LMR 16—A Self-Calibration Procedure for a Leaky Network Analyzer

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Abstract—**A thru-match-reflect/line-match-reflect (TMR/LMR) self-calibration procedure based on the 16-term error model is shown. The error model takes into account all the leakage paths of a wafer prober, test fixture, and network analyzer. Simple closed-form calibration equations are presented. The method is very robust—zero leakage paths and symmetrical or matched-error networks can be handled equally well as more general cases. The algorithm is suitable for nonleaky network analyzers as well. The calibration is comprised of two-port measurement of the following standards: T(L), M–M, R–R, R–M, M–R. Two matched loads (M) are the only standards that have to be known in addition to the thru (T) or line (L). The reflection coefficient of the two identical reflection standards (R) is found in addition to the error parameters as in the normal TMR method. Experimental measurements with the LMR 16 have been made. All the possible combinations of five calibration standards for the 16-term error model are tabulated. The limitations of the super-thru-short-delay algorithm are defined for the first time.**

Index Terms—**Calibration, de-embedding, microwave measurement, network analyzer, scattering parameters, wafer probe.**

I. Introduction

NETWORK-ANALYZER self-calibration procedures for the eight-term error model have been available for over 20 years [1], [2]. A correction procedure [3] is sometimes used to eliminate the switching errors due to a nonideal source and load match. This, together with the isolation measurement, extends the eight-term model essentially to the 12-term error model. After the introduction of the thru-match-reflect (TMR) calibration procedure [4] a larger variety of possible self-calibration techniques have emerged [5]–[7]. Recent research has also been oriented to multiport network analyzers [8]–[10] and to leaky measurement systems modeled by the 16-term error model [11]–[15]. The methods in [11], [14], [15] are iterative numerical procedures. Many times, however, straightforward calibration equations are preferred [12], [13]. There should be less computational effort when closed-form equations are used. In fact, iterative procedures seldomly occur with the eight- and 12-term error models. The only solutions for the 16-term self-calibration are thus far iterative [14], [15], except the one recently published in [16].

While three standards are enough to calibrate the eight- and 12-term error models, at least five two-port standards are needed in conjunction with the 16-term (sometimes called 15-term) model [12]. In this paper, a thru-match-reflect/line-match-reflect-based (TMR/LMR) self-calibration procedure is developed for the 16-term error model. In addition to the normal measurements of T, M–M, and R–R, two additional two-port standards are produced by pairs M–R and R–M (M = match, R = reflect). If there were no leakage paths, the last two measurements would be redundant.

The analysis gives simple closed-form equations for the error terms, for the device-under-test (DUT), and for the unknown reflection coefficient Γ. If a line (delay) is used instead of a zero-length thru, the method gives the ratio Γ/T, where T is $S_{21} = e^{-\gamma l}$ of the line (l is the physical length of the line while γ is the propagation constant). Either T or Γ must be known.

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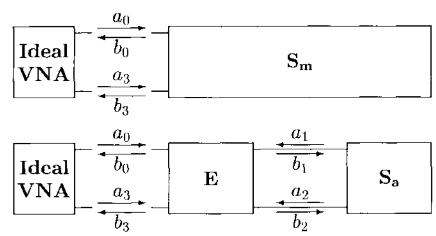

Fig. 1. The measurement configuration. $\mathbf{S_m}$ denotes the measurement raw-data, $\mathbf{S_a}$ being the actual scattering parameters of the standard or DUT, which are measured through the error adapter $\mathbf{E}$.

II. Theory Behind the 16-Term Error Model

A. Formulation

The measurement configuration suitable for the eight- through 16-term error models is shown in Fig. 1. The error network is considered a four-port, as usual [5], [17]–[20]. The port numbering and other conventions are according to [11].

Ports 1 and 2 are the real network analyzer ports directly connected to the DUT denoted by $\mathbf{S_a}$. During the calibration, the standards are measured instead of the DUT. An ideal vector network analyzer (VNA) at ports 0 and 3 sees the DUT through a hypothetical error adapter $\mathbf{E}$. Parameters $\mathbf{S_m}$ are the raw data measured by an ideal network analyzer. The error adapter takes into account most of the nonidealities of the practical network analyzer, wafer-probing system, or test fixture. It is also possible to connect a calibrated nonideal network analyzer to ports 0 and 3 and restrict the error adapter to describe only the rest of the measurement system (two-tier calibration). In this case, the error network is a model of the wafer-probing station.

a_i and b_i are the incident, reflected, or transmitted voltage waves at the input and output terminals. The actual S-parameters $\mathbf{S_a}$ of the DUT or of the calibration standard must fulfill

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \mathbf{S_a} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{a11} & S_{a12} \\ S_{a21} & S_{a22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}. \tag{1}$$

Similarly, the measured (uncorrected) S-parameters $\mathbf{S_m}$ of the DUT or of the calibration standard are equated to the voltage waves as follows:

$$\begin{bmatrix} b_0 \\ b_3 \end{bmatrix} = \mathbf{S_m} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} = \begin{bmatrix} S_{m11} & S_{m12} \\ S_{m21} & S_{m22} \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix}. \tag{2}$$

S-parameters $\mathbf{E}$ and T-parameters $\mathbf{T}$ (transfer or chain scattering matrix) of the error network are defined by

$$\begin{bmatrix} b_0 \\ b_3 \\ b_1 \\ b_2 \end{bmatrix} = \mathbf{E} \begin{bmatrix} a_0 \\ a_3 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \mathbf{E_1} & \mathbf{E_2} \\ \mathbf{E_3} & \mathbf{E_4} \end{bmatrix} \begin{bmatrix} a_0 \\ a_3 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} e_{00} & e_{03} & e_{01} & e_{02} \\ e_{30} & e_{33} & e_{31} & e_{32} \\ e_{10} & e_{13} & e_{11} & e_{12} \\ e_{20} & e_{23} & e_{21} & e_{22} \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} b_0 \\ b_3 \\ a_0 \\ a_3 \end{bmatrix} = \mathbf{T} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \mathbf{T_1} & \mathbf{T_2} \\ \mathbf{T_3} & \mathbf{T_4} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} t_0 & t_1 & t_4 & t_5 \\ t_2 & t_3 & t_6 & t_7 \\ t_8 & t_9 & t_{12} & t_{13} \\ t_{10} & t_{11} & t_{14} & t_{15} \end{bmatrix}. \tag{4}$$

In general, equations between t_{ij}- and e_{ij}-parameters [(5) and (6)] can be found as shown at the bottom of the page, where

$$\Delta_0 = \Delta\mathbf{T_4} = t_{12}t_{15} - t_{13}t_{14} \tag{7}$$
$$\Delta_1 = t_4t_{15} - t_5t_{14} \tag{8}$$
$$\Delta_2 = t_5t_{12} - t_4t_{13} \tag{9}$$
$$\Delta_3 = t_6t_{15} - t_7t_{14} \tag{10}$$
$$\Delta_4 = t_7t_{12} - t_6t_{13} \tag{11}$$
$$\Delta = \Delta\mathbf{E_3} = e_{10}e_{23} - e_{13}e_{20} \tag{12}$$
$$\Delta_5 = e_{13}e_{21} - e_{23}e_{11} \tag{13}$$
$$\Delta_6 = e_{20}e_{11} - e_{10}e_{21} \tag{14}$$
$$\Delta_7 = e_{13}e_{22} - e_{23}e_{12} \tag{15}$$
$$\Delta_8 = e_{20}e_{12} - e_{10}e_{22}. \tag{16}$$

There are an equal number of t_{ij}- and e_{ij}-parameters both with the 8- and 16-term error models. However, in the nine- through 15-term error models there can be more nonzero t_{ij}-parameters than e_{ij}-parameters, making some of the t_{ij}-parameters linearly dependent on each other. Due to the complicated transformation in (5) and (6), the number of t_{ij}-parameters is also dependent on which e_{ij}-parameters are assumed zero (if any). The difference of the number of parameters can be as much as two.

Linear equations for the error terms can be developed by splitting the matrix T into four quadrants [17], [18]

$$\begin{aligned} \begin{bmatrix} b_0 \\ b_3 \end{bmatrix} &= \mathbf{T_1} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \mathbf{T_2} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\ &= (\mathbf{T_1 S_a} + \mathbf{T_2}) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \mathbf{S_m} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} \end{aligned} \tag{17}$$

$$\begin{bmatrix} a_0 \\ a_3 \end{bmatrix} = \mathbf{T_3} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \mathbf{T_4} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = (\mathbf{T_3 S_a} + \mathbf{T_4}) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{18}$$

$$\mathbf{T_1 S_a} + \mathbf{T_2} = \mathbf{S_m}(\mathbf{T_3 S_a} + \mathbf{T_4}) \tag{19}$$
$$\mathbf{S_a} = (\mathbf{T_1} - \mathbf{S_m T_3})^{-1}(\mathbf{S_m T_4} - \mathbf{T_2}). \tag{20}$$

Matrix equation (19) is the basis of the calibration, while (20) allows the de-embedding of the DUT.

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} \mathbf{T_2 T_4^{-1}} & \mathbf{T_1} - \mathbf{T_2 T_4^{-1} T_3} \\ \mathbf{T_4^{-1}} & -\mathbf{T_4^{-1} T_3} \end{bmatrix} \\ &= \frac{1}{\Delta_0} \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_0 t_0 - \Delta_1 t_8 - \Delta_2 t_{10} & \Delta_0 t_1 - \Delta_1 t_9 - \Delta_2 t_{11} \\ \Delta_3 & \Delta_4 & \Delta_0 t_2 - \Delta_3 t_8 - \Delta_4 t_{10} & \Delta_0 t_3 - \Delta_3 t_9 - \Delta_4 t_{11} \\ t_{15} & -t_{13} & t_{10}t_{13} - t_8 t_{15} & t_{13}t_{11} - t_9 t_{15} \\ -t_{14} & t_{12} & t_8 t_{14} - t_{10} t_{12} & t_9 t_{14} - t_{11} t_{12} \end{bmatrix} \end{aligned} \tag{5}$$

$$\begin{aligned} \mathbf{T} &= \begin{bmatrix} \mathbf{E_2} - \mathbf{E_1 E_3^{-1} E_4} & \mathbf{E_1 E_3^{-1}} \\ -\mathbf{E_3^{-1} E_4} & \mathbf{E_3^{-1}} \end{bmatrix} \\ &= \frac{1}{\Delta} \begin{bmatrix} e_{01}\Delta + e_{00}\Delta_5 + e_{03}\Delta_6 & e_{02}\Delta + e_{00}\Delta_7 + e_{03}\Delta_8 & e_{00}e_{23} - e_{03}e_{20} & e_{03}e_{10} - e_{00}e_{13} \\ e_{31}\Delta + e_{30}\Delta_5 + e_{33}\Delta_6 & e_{32}\Delta + e_{30}\Delta_7 + e_{33}\Delta_8 & e_{30}e_{23} - e_{33}e_{20} & e_{33}e_{10} - e_{30}e_{13} \\ \Delta_5 & \Delta_7 & e_{23} & -e_{13} \\ \Delta_6 & \Delta_8 & -e_{20} & e_{10} \end{bmatrix} \end{aligned} \tag{6}$$

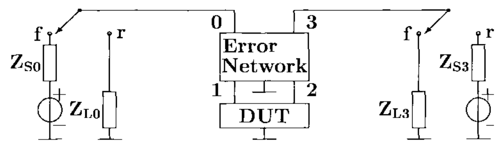

Fig. 2. The effective source and load match in forward (f) and reverse (r) measurements.

These equations can be used with practically any network analyzer, test fixture, or wafer-probing system. Equation (19) produces a set of four linear equations in terms of the 16 error-parameters t_{ij} [12] as follows:

$$\begin{bmatrix} S_{a11} & S_{a21} & & & 1 & & & & -S_{m11}S_{a11} & -S_{m11}S_{a21} & -S_{m12}S_{a11} & -S_{m12}S_{a21} & -S_{m11} & & -S_{m12} & \\ S_{a12} & S_{a22} & & & & 1 & & & -S_{m11}S_{a12} & -S_{m11}S_{a22} & -S_{m12}S_{a12} & -S_{m12}S_{a22} & & -S_{m11} & & -S_{m12} \\ & & S_{a11} & S_{a21} & & & 1 & & -S_{m21}S_{a11} & -S_{m21}S_{a21} & -S_{m22}S_{a11} & -S_{m22}S_{a21} & -S_{m21} & & -S_{m22} & \\ & & S_{a12} & S_{a22} & & & & 1 & -S_{m21}S_{a12} & -S_{m21}S_{a22} & -S_{m22}S_{a12} & -S_{m22}S_{a22} & & -S_{m21} & & -S_{m22} \end{bmatrix} \begin{bmatrix} t_0 \\ t_1 \\ \cdots \\ t_{15} \end{bmatrix} = \mathbf{0}. \qquad \begin{matrix} (21a) \\ (21b) \\ (21c) \\ (21d) \end{matrix}$$

B. Effect of Source and Load Match

In the 12-term error model [21] the reverse and forward-source and load-match terms are defined separately. This is essential with an imperfect measurement system, because the switches change the configuration of the network (Fig. 2).

However, the 16-term error model, is an extension of the 8-term model and by itself cannot take the separate forward- and reverse-load match terms into account. This will decrease the measurement accuracy especially when highly reflective devices are measured [22]. If the forward-source match is not equal to the reverse-load match, or if the forward-load match is not equal to the reverse-source match [see (22)–(26)], will correct the error [3], [15]. It can be shown that the equations are exactly the same as the S-parameters solved from the T-parameters in [5] (10). The equations are used both with the measurement of the calibration standards and with the measurement of the DUT. The following primed quantities denote the reverse measurement direction while the unprimed refer to the forward direction:

$$\begin{aligned} b_0 &= S_{m11}a_0 + S_{m12}a_3 \\ b_0' &= S_{m11}a_0' + S_{m12}a_3' \\ b_3 &= S_{m21}a_0 + S_{m22}a_3 \\ b_3' &= S_{m21}a_0' + S_{m22}a_3' \end{aligned}$$

$$S_{m11} = \frac{\frac{b_0}{a_0} - \frac{b_0'}{a_3'}\frac{a_3}{a_0}}{D} \tag{22}$$

$$S_{m12} = \frac{\frac{b_0'}{a_3'} - \frac{b_0}{a_0}\frac{a_0'}{a_3'}}{D} \tag{23}$$

$$S_{m21} = \frac{\frac{b_3}{a_0} - \frac{b_3'}{a_3'}\frac{a_3}{a_0}}{D} \tag{24}$$

$$S_{m22} = \frac{\frac{b_3'}{a_3'} - \frac{b_3}{a_0}\frac{a_0'}{a_3'}}{D} \tag{25}$$

$$D = 1 - \frac{a_3}{a_0}\frac{a_0'}{a_3'}. \tag{26}$$

This method can only be used with four sampler network analyzers capable of measuring a_0 and a_3 independently. In two-tier calibration with the calibrated network analyzer, the correction is unnecessary.

C. 16-Term De-Embedding Equations

After the error terms are defined, the embedded S-parameters of the DUT $(\mathbf{S_m})$ are first measured. The de-embedding can be performed using the following equations:

$$(\mathbf{S_m T_3} - \mathbf{T_1})\mathbf{S_a} = \mathbf{T_2} - \mathbf{S_m T_4}$$
$$\Leftrightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} S_{a11} & S_{a12} \\ S_{a21} & S_{a22} \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \tag{27}$$

where

$$a = S_{m11}t_8 + S_{m12}t_{10} - t_0 \tag{28}$$
$$b = S_{m11}t_9 + S_{m12}t_{11} - t_1 \tag{29}$$
$$c = S_{m21}t_8 + S_{m22}t_{10} - t_2 \tag{30}$$
$$d = S_{m21}t_9 + S_{m22}t_{11} - t_3 \tag{31}$$
$$e = t_4 - S_{m11}t_{12} - S_{m12}t_{14} \tag{32}$$
$$f = t_5 - S_{m11}t_{13} - S_{m12}t_{15} \tag{33}$$
$$g = t_6 - S_{m21}t_{12} - S_{m22}t_{14} \tag{34}$$
$$h = t_7 - S_{m21}t_{13} - S_{m22}t_{15}. \tag{35}$$

The resulting equations are simply

$$S_{a11} = \frac{de - bg}{ad - bc} \tag{36}$$
$$S_{a12} = \frac{df - bh}{ad - bc} \tag{37}$$
$$S_{a21} = \frac{ag - ec}{ad - bc} \tag{38}$$
$$S_{a22} = \frac{ah - fc}{ad - bc}. \tag{39}$$

D. Comparison with the Eight-Term Transmission-Circuit-Unknown (TCX) Algorithm

Equations (21a)–(21d) are used in the transmission-circuit-unknown (TCX) algorithm [6], [22] without leakage terms. The quadrants $\mathbf{T_i}$ and $\mathbf{E_i}$ are all diagonal matrices in the case of the eight-term error model. If error term t_{12} is set equal to one, t_{15} will correspond to the parameter k in [6] as follows:

$$\mathbf{T} = \begin{bmatrix} -\Delta\mathbf{L} & 0 & L_{11} & 0 \\ 0 & -k\Delta\mathbf{R} & 0 & kR_{11} \\ -L_{22} & 0 & 1 & 0 \\ 0 & -kR_{22} & 0 & k \end{bmatrix} \quad (40)$$

$$\mathbf{E} = \begin{bmatrix} L_{11} & 0 & L_{12} & 0 \\ 0 & R_{11} & 0 & R_{12} \\ L_{21} & 0 & L_{22} & 0 \\ 0 & R_{21} & 0 & R_{22} \end{bmatrix} \quad (41)$$

where $e_{10} = L_{21} = 1$ and $e_{23} = R_{21} = \frac{1}{k}$.

E. Restrictions of the Super-Thru-Short-Delay Algorithm

In [17], [18], the super-thru-short-delay algorithm based on the 16-term error model is presented. In the both articles, it is assumed that one of the quadrants (e.g., $\mathbf{T_1}$) can be arbitrarily chosen. In Speciale and Franzen's preliminary work [17], it is admitted that "no formal proof of this invariance has yet been given." In [18], it is mentioned that some restrictions are expected, but nothing other than singularities are ever stated. If $\mathbf{T_1}$ is assumed known, 12 unknowns are left. Three standards would then possibly be enough for a complete calibration.

Detailed analysis has shown that in its original form, the super-thru-short-delay method gives correct results only under the following special conditions.

- A symmetrical and reciprocal DUT is measured with a measurement system, in which every quadrant $\mathbf{E_i}$ is also symmetrical and reciprocal. An *arbitrary* quadrant $\mathbf{T_1}$ can be chosen as follows:

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}. \quad (42)$$

- Any kind of DUT can be correctly measured if quadrants $\mathbf{E_i}$ are diagonal and symmetrical. This means a symmetrical measurement system with no leakage paths (e.g., a test fixture and two-tier calibration). $\mathbf{T_1}$ has to be diagonal as follows:

$$\mathbf{T_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (43)$$

These conditions can be expressed in an alternative form. The quadrants $\mathbf{E_i}$ must commute with the standards, the DUT, and each other.

In general, three standards allow the measurement of any DUT, if $e_{13} = 0, e_{20} = 0, e_{10} = e_{23} \Rightarrow k = L_{21}/R_{21} = 1$. In practice, these conditions can be encountered only with two-tier calibrations. In fact, the situation is approximately the same as the nearly symmetrical case in [23]. Because three standards instead of two are used, leakage paths other than e_{13} and e_{20} can be included. In [23], inferior results were achieved using the super-thru-short-delay algorithm for thru-short-delay and for thru-open-delay standards, but the results were much better for the thru-short-open combination. The test fixture model was nonleaky and reciprocal, but slightly asymmetrical $(L_{21} \neq R_{21})$. If more leakage paths are assumed to be zero, the value of k can be solved from the data of three calibration measurements.

F. Possible Calibration Methods for the 16-Term Error Model

One of the 16 error terms (in this case t_{12}) is scaled equal to one, in fact, the other terms are calculated as a function of the scaling parameter. At first glance, four calibration measurements seem to give enough equations to solve the remaining 15 error terms. By numerical simulation it can be shown that all the 16 sets of 15 equations are singular for any four standards. The situation does not change even though the standards would be nonreciprocal and nonsymmetrical. This means that five two-port calibration measurements are strictly needed [12]. The only proof of this known to the author is the result of the simulation. Five standards already make the self-calibration possible, although seven were used in [15].

Very good results were achieved using only four calibration measurements in [11]. This is possible if additional assumptions concerning the reciprocity or symmetry of the error network are made, or if at least one of the leakage paths can be neglected (or assumed known). An iterative procedure finds a solution, although the 15 equations are not independent. In practice, this is the same as assuming an arbitrary value to one or more of the unknowns. There is, however, no guarantee that the solution is close to the correct one because the number of possible solutions is infinite.

A sufficient number of independent equations is achieved using thru (T) and/or delay line (D, L) with different combinations of match (M), short (S), or open (O). For example, M–M means matched loads at both ports 1 and 2 simultaneously.

A natural choice for five two-port calibration standards would be pairs M–M, S–S, and O–O, in addition to the thru and the delay line. However, this combination of standards does not allow the determination of the error parameters. The nonsingular combinations verified by careful simulations are listed in Table I. Note, that S and O can be interchanged and T or M–M can always be replaced by L (D). Of course, the left and right ports can also be interchanged. In [12], there were six essentially redundant combinations, which have been omitted here. Complete listings of the 156 different combinations including all the dual cases are shown in [24].

The error parameters can be solved from linear equations produced by (21). A good way is to use (21a)–(21d) for the first and the second standard, (21a)–(21c) for the third standard, and (21b)–(21c) for the two remaining standards. These equations always give a solution for the combinations mentioned above. Yet, the solution is not necessarily always optimal where error sensitivity is concerned.

Five two-port calibration measurements may sound impractical, but taking into account the speed of modern network analyzers it should be no problem. The availability of standards is also usually not a bottleneck, because the same standards are already used in short-open-load-thru (SOLT), line-reflect-line (LRL), and LMR methods. However, two similar one-port standards are needed with many (not all) of the procedures above. Self-calibration procedures usually assume identical reflection-standards on either side. Sexed connectors may cause problems when the same type of standard has to be connected to both analyzer ports, whether it is done simultaneously or not. Wafer prober measurements may, in this respect,

TABLE I
NONSINGULAR COMBINATIONS OF FIVE TWO-PORT CALIBRATION STANDARDS IN CONJUNCTION WITH THE 16-TERM ERROR MODEL. T=THRU, M=MATCH. S=SHORT, O=OPEN OR VICE VERSA. T OR M-M CAN BE REPLACED BY L=LINE (=D=DELAY)

Nr.	St.1	St.2	St.3	St.4	St.5
1	T	S-M	M-S	S-S	O-O
2	T	S-M	O-S	S-S	O-O
3	T	S-M	O-M	S-S	O-O
4	T	S-M	M-S	O-M	S-S
5	T	S-M	M-O	O-M	S-S
6	T	S-M	O-S	O-M	S-S
7	T	S-M	O-S	M-O	S-S
8	T	S-M	O-S	S-O	O-O
9	T	S-M	M-S	S-O	O-O
10	T	S-M	M-S	O-M	S-O
11	T	S-M	M-S	O-M	O-S
12	T	S-M	M-O	S-O	O-S
13	T	S-M	O-M	S-O	O-S
14	T	M-M	S-S	O-O	S-O
15	T	M-M	S-S	O-O	S-M
16	T	M-M	S-S	S-O	O-S
17	T	M-M	S-S	S-O	M-S
18	T	M-M	S-S	S-O	M-O
19	T	M-M	S-S	S-M	M-S
20	T	M-M	S-S	S-M	M-O
21	T	M-M	S-S	O-M	M-O
22	T	M-M	S-O	S-M	O-S
23	T	M-M	S-O	S-M	M-S
24	T	M-M	S-O	S-M	M-O
25	T	M-M	S-O	O-M	M-S
26	T	D	S-S	S-O	M-M
27	T	D	S-S	S-M	M-M
28	T	D	S-S	O-M	M-M
29	T	D	S-O	S-M	M-M
30	T	D	S-O	M-S	M-M
31	T	D	S-M	M-S	M-M
32	T	D	S-M	M-O	M-M
33	T	D	S-S	S-O	O-M
34	T	D	S-S	S-M	O-M
35	T	D	S-O	S-M	O-M
36	T	D	S-M	M-S	O-M

be easier to accomplish than calibration at the coaxial ports (if sexed). In general, the one-port standard at port 1 does not have to be identical to the same type of standard (short, open, or match) at port 2, if the standards are exactly known. It is, however, expected that several self-calibration procedures are feasible using the standard combinations in Table I. The combination Nr. 14 is already used in [16] as a self-calibration procedure. Another one will follow in the next section.

III. SELF-CALIBRATION PROCEDURE LMR 16

The following procedure allows 16-term calibration using standards T, M–M, R–M, R–R, and M–R (Nr. 19 in Table I). T means a thru or a delay (line), M is an ideal match, and R is an unknown reflection standard (typically nonideal or ideal short or open). Measuring the standards in this order necessitates only six connections in the sense of [15] in comparison with seven and five connections needed with the 12-term thru-reflect-line (TRL) or TMR, respectively. In this case it is not possible to replace M–M with a delay line.

Five two-port measurements are made with the standards $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{E}$, each representing S-parameters $\mathbf{S_a}$ in Fig. 1. The measurement data corresponding to $\mathbf{S_m}$ are $\mathbf{M_A}$, $\mathbf{M_B}$, $\mathbf{M_C}$, $\mathbf{M_D}$, and $\mathbf{M_E}$, respectively. Assuming an ideal match, S-parameters of the standards will be

$$\text{thru or delay} \quad \mathbf{A} = \begin{bmatrix} 0 & T \\ T & 0 \end{bmatrix} \tag{44}$$

$$\text{match–match} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{45}$$

$$\text{reflect–reflect} \quad \mathbf{C} = \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix} \tag{46}$$

$$\text{reflect–match} \quad \mathbf{D} = \begin{bmatrix} \Gamma & 0 \\ 0 & 0 \end{bmatrix} \tag{47}$$

$$\text{match–reflect} \quad \mathbf{E} = \begin{bmatrix} 0 & 0 \\ 0 & \Gamma \end{bmatrix} \tag{48}$$

where $T = e^{-\gamma l}$, l and γ are the physical length and the propagation constant of the line. For a zero-length thru $T = 1$. Because the leakage affects the measurement, the one-port standards have to be simultaneously connected to the analyzer ports in measurements $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{E}$. If no leakage paths are present, standards $\mathbf{D}$ and $\mathbf{E}$ are redundant. The same data is included in $\mathbf{B}$ and $\mathbf{C}$.

In the course of the calibration, the value of the reflection coefficient Γ can be found from a second-order equation as in the eight-term LMR method. In fact, the unknown is $\frac{\Gamma}{T}$ so that T can be calculated if Γ is known or vice versa. Of course, neither Γ nor T is allowed to be equal to zero. The resulting equations are surprisingly simple, partly because of the excessive use of the match as a standard.

To improve readability, the resulting equations are shown first. The derivation of the method is outlined in the Appendix.

Matrices $\mathbf{M}$, $\mathbf{N}$, $\mathbf{O}$, $\mathbf{P}$, and $\mathbf{R}$ are based on the measurement data and defined by

$$\mathbf{M} = (\mathbf{M_A} - \mathbf{M_C})\mathbf{N} \tag{49}$$

$$\mathbf{N} = (\mathbf{M_E} - \mathbf{M_A})^{-1}(\mathbf{M_B} - \mathbf{M_E}) \tag{50}$$

$$\mathbf{O} = \mathbf{M_B} - \mathbf{M_C} \tag{51}$$

$$\mathbf{P} = (\mathbf{M_A} - \mathbf{M_C})\mathbf{R} \tag{52}$$

$$\mathbf{R} = (\mathbf{M_D} - \mathbf{M_A})^{-1}(\mathbf{M_B} - \mathbf{M_D}). \tag{53}$$

Coefficients m, n, o, and p are functions of the elements of the matrices

$$m = (P_{21} + O_{21})M_{22} - (P_{22} + O_{22})M_{21} \tag{54}$$

$$n = O_{21}P_{12} - O_{22}P_{11} \tag{55}$$

$$o = (M_{12} + O_{12})P_{11} - (M_{11} + O_{11})P_{12} \tag{56}$$

$$p = O_{12}M_{21} - O_{11}M_{22}. \tag{57}$$

As mentioned earlier, t_{12} is used as a scaling parameter and is assumed to be equal to one. Either Γ/T or t_{15} has to be calculated from a second-order equation

$$t_{12} = 1 \tag{58}$$

$$\left(\frac{\Gamma}{T}\right)^2 = \frac{mo}{np} \quad (t_{15})^2 = \frac{mp}{no}\frac{P_{11}^2}{M_{22}^2}t_{12}^2 \tag{59}$$

$$t_{15} = -\frac{p}{o}\frac{P_{11}}{M_{22}}\frac{\Gamma}{T}t_{12} \quad \frac{\Gamma}{T} = -\frac{o}{p}\frac{M_{22}}{P_{11}}\frac{t_{15}}{t_{12}} \tag{60}$$

$$t_{13} = -\frac{P_{12}}{P_{11}}t_{15} \tag{61}$$

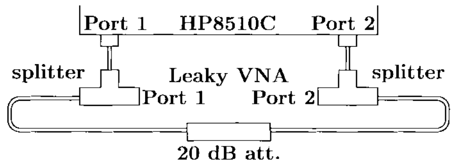

Fig. 3. An artificial leaky VNA.

$$t_{14} = -\frac{M_{21}}{M_{22}} t_{12}. \tag{62}$$

The problem of root choice is solved as usual—the reflection coefficient is approximately known (e.g., $+1$ or -1) or the delay-line length is known with an accuracy of better than $\pm\lambda/4$ (λ = wavelength). The root can possibly be chosen based on the fact that $k = t_{15}/t_{12} \approx +1$ with port-to-port symmetrical test fixtures or wafer-probing stations in two-tier calibration [6].

The remaining error terms can be easily calculated as follows:

$$t_8 = (R_{11}t_{12} + R_{12}t_{14})\frac{1}{\Gamma} - t_{13}\frac{1}{T} \tag{63}$$
$$t_9 = (N_{11}t_{13} + N_{12}t_{15})\frac{1}{\Gamma} - t_{12}\frac{1}{T} \tag{64}$$
$$t_{10} = (R_{21}t_{12} + R_{22}t_{14})\frac{1}{\Gamma} - t_{15}\frac{1}{T} \tag{65}$$
$$t_{11} = (N_{21}t_{13} + N_{22}t_{15})\frac{1}{\Gamma} - t_{14}\frac{1}{T} \tag{66}$$
$$t_0 = M_{C11}t_8 + M_{C12}t_{10} - \frac{1}{\Gamma}(O_{11}t_{12} + O_{12}t_{14}) \tag{67}$$
$$t_1 = M_{C11}t_9 + M_{C12}t_{11} - \frac{1}{\Gamma}(O_{11}t_{13} + O_{12}t_{15}) \tag{68}$$
$$t_2 = M_{C21}t_8 + M_{C22}t_{10} - \frac{1}{\Gamma}(O_{21}t_{12} + O_{22}t_{14}) \tag{69}$$
$$t_3 = M_{C21}t_9 + M_{C22}t_{11} - \frac{1}{\Gamma}(O_{21}t_{13} + O_{22}t_{15}) \tag{70}$$
$$t_4 = M_{B11}t_{12} + M_{B12}t_{14} \tag{71}$$
$$t_5 = M_{B11}t_{13} + M_{B12}t_{15} \tag{72}$$
$$t_6 = M_{B21}t_{12} + M_{B22}t_{14} \tag{73}$$
$$t_7 = M_{B21}t_{13} + M_{B22}t_{15}. \tag{74}$$

Additionally, the source- and load-match correction can be used during the measurements.

IV. Practical Measurements

The measurement network shown in Fig. 3 was used to test the new algorithm in practice. The configuration can already be considered a de facto standard being used by several authors [11], [13], [15].

A weak leakage path was produced connecting a 20-dB attenuator between the third ports of two Wilkinson power dividers (center frequency 1.4 GHz). The connector type used was SMA. The LMR-16 calibration was performed with male and female shielded opens as (∼ identical) reflection standards and fixed loads as match standards. Measurements revealed differences between the two reflection standards due to connector wear. The source and load match correction was not used.

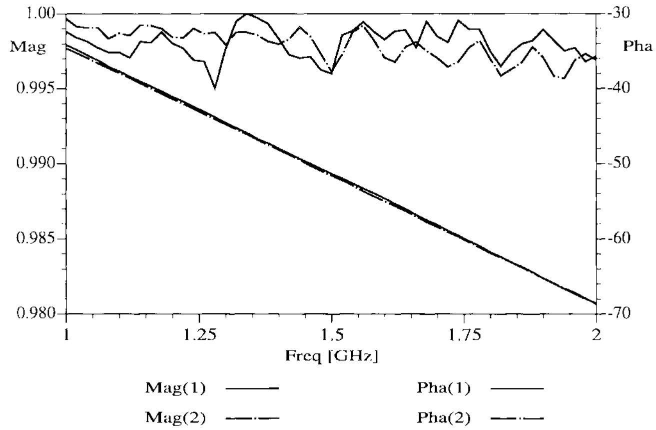

Fig. 4. S_{12} of a delay line. Curve 1: LMR 16 with the leaky network analyzer. Curve 2: Direct measurement with HP8510C.

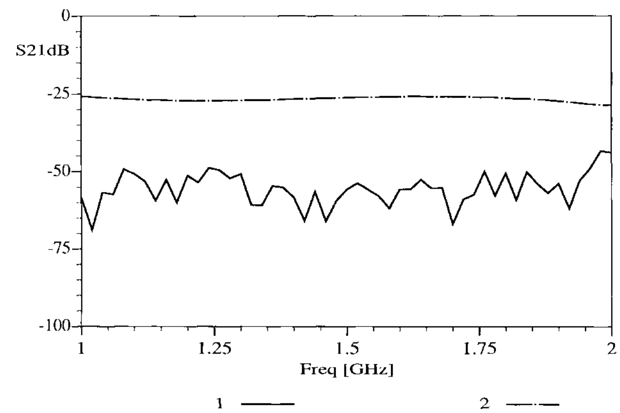

Fig. 5. Isolation measurement with shorts connected to the leaky VNA ports. Curve 1: LMR 16 with the leaky network analyzer. Curve 2: S_{21} seen by the calibrated HP8510C when the leaky VNA with shorts was measured.

As an example, S_{12} of a delay line is shown in Fig. 4. The results of the new LMR-16 calibration method are compared with the direct HP8510C network analyzer measurement (51 frequency points, 12-term SOLT).

To illustrate that the new procedure is able to handle the leakage, isolation measurements were done with short circuits at the measurement ports. Fig. 5 compares the 16-term corrected results to the isolation seen by the network analyzer. It is important that calibration standards are not used any more as test objects when the accuracy of the calibration is checked. As a result, one would get the originally assumed S-parameters of the calibration standards. When measuring the isolation of the termination pairs M–M, O–O, O–M, and M–O (which were used in calibration), nearly ideal values were achieved as expected.

The measurement accuracy can still be enhanced by using identical reflection standards at both ports. In numerical simulations, no roundoff or systematical errors were found. A normal nonleaky network analyzer alone can also be calibrated with the method. As an extreme special case, the algorithm also works with totally ideal error networks, i.e., the calibration standards are measured with a calibrated network analyzer. In such a case, the error networks reduce to two matched zero-length lines with no interconnection.

V. Conclusion

A new class of calibration procedures especially suitable for network analyzers and wafer probes is introduced. The procedures rely on five two-port calibration measurements, which is the minimum number needed for full determination of the error coefficients with the 16-term error model. Possible combinations of thru, delay, match, short, and open are tabulated. The theory of the 16-term error model, including the calibration and de-embedding equations, is outlined. The restrictions of the 16-term methods published in earlier literature are defined. Equations for the LMR 16 self-calibration procedure are given. The usability of the algorithm is shown through measurement examples.

Appendix

The derivation of the self-calibration procedure is given here in detail. Only the most common matrix operations are needed. No advantage was found when formulating the problem with Kronecker products [18], which in this case lead to 4 × 4 matrices. On the contrary, not bigger than 2 × 2 matrices have to be inverted with the present method.

Unity matrix $\mathbf{I}$ and matrices $\mathbf{Q}$, $\mathbf{N_e}$, $\mathbf{S_e}$, $\mathbf{S_w}$, and $\mathbf{N_w}$ (cf. north–west) are denoted as follows:

$$\mathbf{N_w} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathbf{N_e} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{A1}$$

$$\mathbf{S_w} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{S_e} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{A2}$$

$$\mathbf{Q} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{A3}$$

S-parameters of the calibration standards

$$\text{thru or delay} \quad \mathbf{A} = T\mathbf{Q} \tag{A4}$$

$$\text{match–match} \quad \mathbf{B} = \mathbf{0} \tag{A5}$$

$$\text{reflect–reflect} \quad \mathbf{C} = \Gamma\mathbf{I} \tag{A6}$$

$$\text{reflect–match} \quad \mathbf{D} = \Gamma\mathbf{N_w} \tag{A7}$$

$$\text{match–reflect} \quad \mathbf{E} = \Gamma\mathbf{S_e}. \tag{A8}$$

The following five matrix equations are written using (19):

$$\mathbf{M_A}(\mathbf{T_3A} + \mathbf{T_4}) - \mathbf{T_1A} = \mathbf{T_2} \tag{A9}$$

$$\mathbf{M_B}(\mathbf{T_3B} + \mathbf{T_4}) - \mathbf{T_1B} = \mathbf{T_2} \tag{A10}$$

$$\mathbf{M_C}(\mathbf{T_3C} + \mathbf{T_4}) - \mathbf{T_1C} = \mathbf{T_2} \tag{A11}$$

$$\mathbf{M_D}(\mathbf{T_3D} + \mathbf{T_4}) - \mathbf{T_1D} = \mathbf{T_2} \tag{A12}$$

$$\mathbf{M_E}(\mathbf{T_3E} + \mathbf{T_4}) - \mathbf{T_1E} = \mathbf{T_2}. \tag{A13}$$

By eliminating $\mathbf{T_2}$

$$\mathbf{M_BT_3B} - \mathbf{M_AT_3A} + (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4} = \mathbf{T_1}(\mathbf{B} - \mathbf{A})$$
$$\mathbf{M_BT_3B} - \mathbf{M_CT_3C} + (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4} = \mathbf{T_1}(\mathbf{B} - \mathbf{C})$$
$$\mathbf{M_AT_3A} - \mathbf{M_DT_3D} + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4} = \mathbf{T_1}(\mathbf{A} - \mathbf{D})$$
$$\mathbf{M_AT_3A} - \mathbf{M_ET_3E} + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4} = \mathbf{T_1}(\mathbf{A} - \mathbf{E})$$

we get the following equations for $\mathbf{T_1}$:

$$[\mathbf{M_BT_3B} - \mathbf{M_AT_3A} + (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4}](\mathbf{B} - \mathbf{A})^{-1} = \mathbf{T_1} \tag{A14}$$

$$[\mathbf{M_BT_3B} - \mathbf{M_CT_3C} + (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4}](\mathbf{B} - \mathbf{C})^{-1} = \mathbf{T_1} \tag{A15}$$

$$[\mathbf{M_AT_3A} - \mathbf{M_DT_3D} + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4}](\mathbf{A} - \mathbf{D})^{-1} = \mathbf{T_1} \tag{A16}$$

$$[\mathbf{M_AT_3A} - \mathbf{M_ET_3E} + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4}](\mathbf{A} - \mathbf{E})^{-1} = \mathbf{T_1}. \tag{A17}$$

Finally, eliminating $\mathbf{T_1}$

$$\begin{aligned}&[\mathbf{M_BT_3B} - \mathbf{M_AT_3A} + (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4}](\mathbf{B} - \mathbf{A})^{-1} \\ &\quad = [\mathbf{M_BT_3B} - \mathbf{M_CT_3C} + (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4}](\mathbf{B} - \mathbf{C})^{-1}\end{aligned} \tag{A18}$$

$$\begin{aligned}&[\mathbf{M_BT_3B} - \mathbf{M_AT_3A} + (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4}](\mathbf{B} - \mathbf{A})^{-1} \\ &\quad = [\mathbf{M_AT_3A} - \mathbf{M_DT_3D} + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4}](\mathbf{A} - \mathbf{D})^{-1}\end{aligned} \tag{A19}$$

$$\begin{aligned}&[\mathbf{M_BT_3B} - \mathbf{M_AT_3A} + (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4}](\mathbf{B} - \mathbf{A})^{-1} \\ &\quad = [\mathbf{M_AT_3A} - \mathbf{M_ET_3E} + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4}](\mathbf{A} - \mathbf{E})^{-1}.\end{aligned} \tag{A20}$$

Substituting the standards' S-matrices into the previous equations

$$\begin{aligned}&\mathbf{M_AT_3} - (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4Q}\frac{1}{T} \\ &\quad = \mathbf{M_CT_3} - (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4}\frac{1}{\Gamma}\end{aligned} \tag{A21}$$

$$\begin{aligned}&\mathbf{M_AT_3} - (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4Q}\frac{1}{T} \\ &\quad = \mathbf{M_AT_3}\left(\mathbf{I} + \mathbf{N_e}\frac{\Gamma}{T}\right) - \mathbf{M_DT_3N_e}\frac{\Gamma}{T} \\ &\qquad + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4}\left(\mathbf{Q}\frac{1}{T} + \mathbf{S_e}\frac{\Gamma}{T^2}\right)\end{aligned} \tag{A22}$$

$$\begin{aligned}&\mathbf{M_AT_3} - (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4Q}\frac{1}{T} \\ &\quad = \mathbf{M_AT_3}\left(\mathbf{I} + \mathbf{S_w}\frac{\Gamma}{T}\right) - \mathbf{M_ET_3S_w}\frac{\Gamma}{T} \\ &\qquad + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4}\left(\mathbf{Q}\frac{1}{T} + \mathbf{N_w}\frac{\Gamma}{T^2}\right).\end{aligned} \tag{A23}$$

Next, (A22) and (A23) are multiplied by T/Γ, which after some manipulation yields

$$\begin{aligned}(\mathbf{M_A} - \mathbf{M_C})\mathbf{T_3} &= (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4Q}\frac{\mathbf{1}}{\mathbf{T}} \\ &\quad - (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4}\frac{1}{\Gamma}\end{aligned} \tag{A24}$$

$$\begin{aligned}(\mathbf{M_D} - \mathbf{M_A})\mathbf{T_3N_e} &= (\mathbf{M_B} - \mathbf{M_D})\mathbf{T_4Q}\frac{\mathbf{1}}{\mathbf{\Gamma}} \\ &\quad + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4S_e}\frac{1}{T}\end{aligned} \tag{A25}$$

$$\begin{aligned}(\mathbf{M_E} - \mathbf{M_A})\mathbf{T_3S_w} &= (\mathbf{M_B} - \mathbf{M_E})\mathbf{T_4Q}\frac{\mathbf{1}}{\mathbf{\Gamma}} \\ &\quad + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4N_w}\frac{1}{T}.\end{aligned} \tag{A26}$$

It is preferable to solve $\mathbf{T_3}$ as a function of $\mathbf{T_4}$ and not vice versa to maintain the analogy to the TCX algorithm. Yet the inverse of the matrix $(\mathbf{M_A} - \mathbf{M_C})$ does not always exist. To overcome this problem a strange approach is used.

By multiplying (A24) first by $\mathbf{N_e}$ and then by $\mathbf{S_w}$, both from the right-hand side (RHS), and noting that $\mathbf{QN_e} = \mathbf{S_e}$ and $\mathbf{QS_w} = \mathbf{N_w}$, one gets

$$(\mathbf{M_A} - \mathbf{M_C})\mathbf{T_3N_e} = (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4S_e}\frac{1}{\mathbf{T}} - (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4N_e}\frac{1}{\Gamma} \quad \text{(A27)}$$

$$(\mathbf{M_A} - \mathbf{M_C})\mathbf{T_3S_w} = (\mathbf{M_B} - \mathbf{M_A})\mathbf{T_4N_w}\frac{1}{\mathbf{T}} - (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4S_w}\frac{1}{\Gamma}. \quad \text{(A28)}$$

After solving $\mathbf{T_3N_e}$ and $\mathbf{T_3S_w}$ from (A25) and (A26)

$$\mathbf{T_3N_e} = (\mathbf{M_D} - \mathbf{M_A})^{-1}\left[(\mathbf{M_B} - \mathbf{M_D})\mathbf{T_4Q}\frac{1}{\Gamma} + (\mathbf{M_A} - \mathbf{M_D})\mathbf{T_4S_e}\frac{1}{T}\right] \quad \text{(A29)}$$

$$\mathbf{T_3S_w} = (\mathbf{M_E} - \mathbf{M_A})^{-1}\left[(\mathbf{M_B} - \mathbf{M_E})\mathbf{T_4Q}\frac{1}{\Gamma} + (\mathbf{M_A} - \mathbf{M_E})\mathbf{T_4N_w}\frac{1}{T}\right] \quad \text{(A30)}$$

and combining with (A27) and (A28)

$$(\mathbf{M_A} - \mathbf{M_C})(\mathbf{M_D} - \mathbf{M_A})^{-1}(\mathbf{M_B} - \mathbf{M_D})\mathbf{T_4Q} + (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4N_e} + (\mathbf{M_C} - \mathbf{M_B})\mathbf{T_4S_e}\frac{\Gamma}{T} = \mathbf{0} \quad \text{(A31)}$$

$$(\mathbf{M_A} - \mathbf{M_C})(\mathbf{M_E} - \mathbf{M_A})^{-1}(\mathbf{M_B} - \mathbf{M_E})\mathbf{T_4Q} + (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4S_w} + (\mathbf{M_C} - \mathbf{M_B})\mathbf{T_4N_w}\frac{\Gamma}{T} = \mathbf{0}. \quad \text{(A32)}$$

Using a more compact notation for (A31)–(A32)

$$\mathbf{PT_4Q} + \mathbf{OT_4N_e} - \mathbf{OT_4S_e}\frac{\Gamma}{T} = \mathbf{0} \quad \text{(A33)}$$

$$\mathbf{MT_4Q} + \mathbf{OT_4S_w} - \mathbf{OT_4N_w}\frac{\Gamma}{T} = \mathbf{0}. \quad \text{(A34)}$$

$\mathbf{M}$, $\mathbf{O}$, and $\mathbf{P}$ are defined in (49)–(53). $\mathbf{T_4}$ contains three unknown error terms, if t_{12} is set, equal to one of the following:

$$\mathbf{T_4} = \begin{bmatrix} t_{12} & t_{13} \\ t_{14} & t_{15} \end{bmatrix}. \quad \text{(A35)}$$

Both (A33) and (A34) give four equations for the unknown error terms

$$P_{11}t_{13} + P_{12}t_{15} = 0 \quad \text{(A36)}$$

$$P_{11}t_{12} + P_{12}t_{14} + O_{11}\left(t_{12} - \frac{\Gamma}{T}t_{13}\right) + O_{12}\left(t_{14} - \frac{\Gamma}{T}t_{15}\right) = 0 \quad \text{(A37)}$$

$$P_{21}t_{13} + P_{22}t_{15} = 0 \quad \text{(A38)}$$

$$P_{21}t_{12} + P_{22}t_{14} + O_{21}\left(t_{12} - \frac{\Gamma}{T}t_{13}\right) + O_{22}\left(t_{14} - \frac{\Gamma}{T}t_{15}\right) = 0 \quad \text{(A39)}$$

$$M_{11}t_{13} + M_{12}t_{15} + O_{11}\left(t_{13} - \frac{\Gamma}{T}t_{12}\right) + O_{12}\left(t_{15} - \frac{\Gamma}{T}t_{14}\right) = 0 \quad \text{(A40)}$$

$$M_{11}t_{12} + M_{12}t_{14} = 0 \quad \text{(A41)}$$

$$M_{21}t_{13} + M_{22}t_{15} + O_{21}\left(t_{13} - \frac{\Gamma}{T}t_{12}\right) + O_{22}\left(t_{15} - \frac{\Gamma}{T}t_{14}\right) = 0 \quad \text{(A42)}$$

$$M_{21}t_{12} + M_{22}t_{14} = 0. \quad \text{(A43)}$$

One of the error terms t_{14} can already be solved using (A43). Expressing t_{13} as a function of t_{15} from (A36) and substituting into (A39)–(A40)

$$t_{14} = -\frac{M_{21}}{M_{22}}t_{12} \quad \text{(A44)}$$

$$t_{13} = -\frac{P_{12}}{P_{11}}t_{15} \quad \text{(A45)}$$

$$P_{21}t_{12} - P_{22}\frac{M_{21}}{M_{22}}t_{12} + O_{21}t_{12} + O_{21}\frac{P_{12}}{P_{11}}t_{15}\frac{\Gamma}{T} - O_{22}\frac{M_{21}}{M_{22}}t_{12} - O_{22}t_{15}\frac{\Gamma}{T} = 0 \quad \text{(A46)}$$

$$M_{12}t_{15} - M_{11}\frac{P_{12}}{P_{11}}t_{15} + O_{12}t_{15} + O_{12}\frac{M_{21}}{M_{22}}t_{12}\frac{\Gamma}{T} - O_{11}\frac{P_{12}}{P_{11}}t_{15} - O_{11}t_{12}\frac{\Gamma}{T} = 0. \quad \text{(A47)}$$

Error term t_{15} and ratio Γ/T can be solved simultaneously from (A46) and (A47). Once $\mathbf{T_4}$ is determined, $\mathbf{T_3}$ can be found from (A29) to (A30)

$$\mathbf{T_3N_e} = \begin{bmatrix} 0 & t_8 \\ 0 & t_{10} \end{bmatrix} = \mathbf{RT_4Q}\frac{1}{\Gamma} - \mathbf{T_4S_e}\frac{1}{T}$$

$$\mathbf{T_3S_w} = \begin{bmatrix} t_9 & 0 \\ t_{11} & 0 \end{bmatrix} = \mathbf{NT_4Q}\frac{1}{\Gamma} - \mathbf{T_4N_w}\frac{1}{T}.$$

$\mathbf{R}$ and $\mathbf{N}$ are defined in (49)–(53). With the calibration standards defined above, (A15) and (A10) will reduce to

$$\mathbf{T_1} = \mathbf{M_CT_3} - (\mathbf{M_B} - \mathbf{M_C})\mathbf{T_4}\frac{1}{\Gamma}$$

$$\mathbf{T_2} = \mathbf{M_BT_4}.$$

The eight remaining error terms can then finally be solved.

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